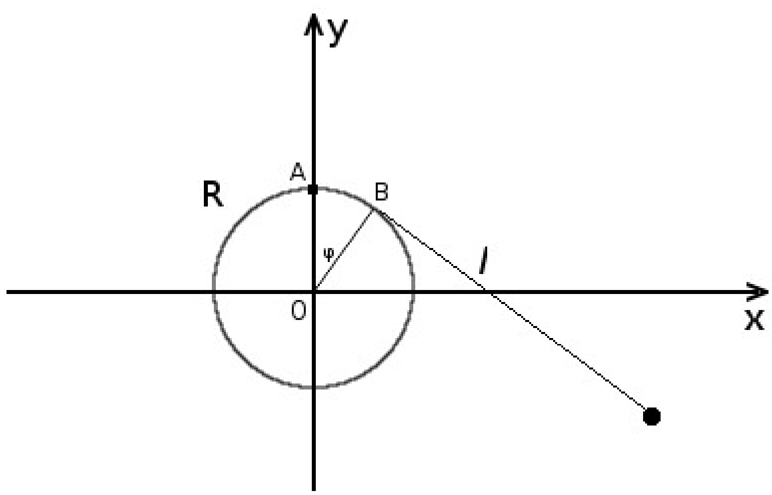
Sample math exercises for students taking Physics 1600/1610 at Auburn University

Algebra

1. a) Solve the equation $x^2 - kx - 6k^2 = 0$ with respect to x, where k is a real number.
   b) Now consider the equation $x^2 - kx - 6k^2 = a$ where a is also a real number. For what values of a does this equation have no real solutions?
2. Simplify the expression
   $$\frac{m_1 v_1^2}{2}+\frac{m_2 v_2^2}{2}-\frac{m_1+m_2}{2}\left(\frac{m_1 v_1+m_2 v_2}{m_1+m_2}\right)^2$$
3. Solve the system of equations with respect to x and y:
   $$\begin{cases} k x+a y=2k^2 \\ 2k x+b y=3k^2 \end{cases}$$
   What should the relationship between a and b be so that the system would have no solution (the equations would be incompatible)?
4. Solve the system of equations with respect to x and y:
   $$\begin{cases} a x+y=\frac{1}{a} \\ x y=a \end{cases}$$

Geometry



In the setup above, a string of length l is attached to the top point A of a cylinder with the radius R. On the other end of the string there is a ball. The string touches the cylinder along the arc from point A to point B, whose angle is φ, and then goes straight, tangent to the cylinder's surface. The string and ball are on the xy-plane whose origin is at the center of the circular cross-section of the cylinder. Find the x and y coordinates of the ball in terms of R, l, and φ.

(Useful questions to ask yourself first: What length of the string touches the cylinder? What length of the string is in the air? What are the coordinates of point B?)

Trigonometry

1. Using the half-argument identities, find the *exact* (not using a calculator, which gives you an approximate value) values of the following functions:
   sin 15º; cos 15º; sin 75º; cos 22.5º.
2. Prove the identity:
   $$\frac{\cos\beta}{\cos\alpha+\sin\beta}+\frac{\sin\beta}{\cos\beta+\sin\alpha}=\frac{1+\sin(\alpha+\beta)}{\cos(\alpha-\beta)+\frac{1}{2}(\sin 2\alpha+\sin 2\beta)}$$
3. Derive the triple-argument identities for the *sin* and *cos* functions, i. e. prove that
   $\sin 3\alpha=3\sin\alpha-4\sin^3\alpha$, $\cos 3\alpha=4\cos^3\alpha-3\cos\alpha$
4. Solve the equation sin x + cos x = a for x on the interval [0, π/2].