One-Dimensional Motion

The position of the particle on the one-dimensional axis can change with time. Therefore, there exists a function that determines the position at any moment of time. (If you need more information about the term "function", see http://en.wikipedia.org/wiki/Function_%28mathematics%29.) We call it the position function. If the position axis is x and time is t, the position function will be $x(t)$.

Its first and second time derivatives are also functions with well-known names. $\frac{dx}{dt} \stackrel{\text{def}}{=} v$ is called *speed*, $\frac{d^2x}{dt^2} = \frac{dv}{dt} \stackrel{\text{def}}{=} a$ is called *acceleration*.

Here we will study only three types of motion: x = const, v = const, a = const.

1. ***x* = const.** It is clear that any derivative of x will be zero. Therefore, $v = 0$, $a = 0$. The particle is not changing its position, i.e. it is at rest at all times. This case corresponds to **no motion**; the particle is **resting**.
2. ***v* = const.** Acceleration is therefore zero, but the position function is linear:

$$x = \int v\,dt = v \int dt = vt + x_0 \tag{1}$$

   where x_0 is the constant of integration corresponding to the particle's initial position. This case is called **uniform motion.**
3. ***a* = const.** Integrating up to v and further to x, we have

$$v = \int a\,dt = a \int dt = at + v_0 \tag{2}$$

$$x = \int v\,dt = \int (at + v_0)\,dt = \int at\,dt + \int v_0\,dt = a \int t\,dt + v_0 \int dt = \frac{at^2}{2} + v_0 t + x_0 \tag{3}$$

   This case is called accelerated motion and the speed and position functions for it are often called the basic kinematic equations, since accelerated motion is extremely widespread in kinematics.

$$v = v_0 + at \tag{4}$$

$$x = v_0 t + \frac{at^2}{2} \tag{5}$$

   In the last one the initial position x_0 is omitted because it is almost always convenient to put the zero of your coordinate into the place where motion begins, thus making $x_0 = 0$.

Average speed on a certain interval of motion is defined as the ratio of the length of the interval and the time during which it was covered by motion:

$$\langle v\rangle=\frac{x_{total}}{t_{total}} \tag{6}$$

Example problem. A car driver traveling at speed v_0 saw a speed bump from the distance s and applied the brakes. He decelerated with the magnitude a. At what speed did he go over the bump?

Solution. This is accelerated motion. Putting the origin at the place where he saw the bump (and thus making the initial position zero), we write the kinematic equations:

$$v=v_0-at,\ \ s=v_0t-\frac{at^2}{2} \tag{7}$$

Notice the minus sign (if a is the *magnitude*, a positive quantity, you must not forget the minus if the driver *decreased* the speed). This is a system of 2 equations with 2 unknowns: t, the time of motion from the origin to the bump, and v, the speed which we are looking for. Solving the 2nd equation for t (it is quadratic)

$$\frac{a}{2}t^2-v_0t+s=0;\ \ t=\frac{v_0\pm\sqrt{v_0^2-2as}}{a} \tag{8}$$

and substituting t into the first equation, which is already a ready-made solution for v, we get

$$v=\sqrt{v_0^2-2as} \tag{9}$$

We had to throw out the solution with the plus sign because it gave us a negative value and thus did not correspond to the situation. Also notice that v_0^2-2as must be positive, otherwise the driver will stop before the bump is reached.

Additional question. What is the average speed of the car on the whole distance to the bump?
Answer. Average speed is total distance over total time. In this case, the distance is s and the time is

$$t=\frac{v_0-\sqrt{v_0^2-2as}}{a} \tag{10}$$

so the average speed is

$$\langle v\rangle=\frac{as}{v_0-\sqrt{v_0^2-2as}} \tag{11}$$